Optimal Selection of Advertising Print-Media using Linear Programming

Thaw Tar Aye, Myint Myint Yee
University of Computer Studies, Yangon, Myanmar
thawtarsoenyunt@gmail.com

Abstract

Operation Research (OR) is an interdisciplinary branch of applied mathematics and formal science that uses method like mathematical modeling, statistics and algorithms to arrive at optimal or near optimal solution to complex problem. Linear Programming is widely used model type that can solve decision problem with thousands of variables in OR. The BIG-M method has approved to be the most effective technique applicable to any problem in Linear Programming, and can be formulated in term of linear objective function, subject to a set of constraints. BIG-M can be used to help marketing manager allocate a fixed budget to various advertising media. In this paper, we analyzed the basic concepts of Linear Programming and BIG-M method, and applied BIG-M method with Simplex algorithm in advertising media selection problem of marketing. The system produces the optimal advertising media and advertisement times using BIG-M method with Simplex algorithm according to target budget and audience constraints. The implemented system is intended to help generic business to find out the efficient advertising media and expense for their market.

1. Introduction

Many systems in business and economics involve a process called optimization, in which required finding the minimum cost, the maximum profit, or the minimum use of resources. In marketing, the marketers need to determine the right mix of media exposure to use in an advertising campaign. The goal is to determine how many advertisements to place in each medium where the cost of placing the advertisement depends on the medium and to minimize the total cost of the advertising campaign, but subject to a series of constraints.

Each medium may provide only certain exposure of the target audience; there may be a lower bound on the total exposure from the campaign. Plus there may be limit on the availability of each medium for an advertising platform. Today, Linear Programming is used to model successfully numerous real world situations, ranging from scheduling airline routes, to shipping oil from refineries to cities, to find inexpensive diets capable of meeting the minimum daily requirements. It is a standard tool that has saved expense in marketing for most companies or businesses in the various industrialized countries in the world.

This paper implemented the media selection system that help the business to find out the best advertising media according to constraint set implemented by BIG-M method.

2. Linear Programming

Linear Programming (LP) is used to find the best or optimal solution to a problem that requires a decision or set of decisions about how best to use a set of limited resource to achieve a state goal of objectives. Linear Programming requires that all the mathematical functions in the model be linear functions. It is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Linear Programming problem is described using the standard form. It consists of the following three parts:

Non-Negative Variable: The values of the variables are not known when starting the problem. The variables usually represent things that can adjust or control. The goal is to find values of the variables that provide the best value of the objective function.

$$x_1 \geq 0 \tag{1}$$

Where, x_1, x_2 are the variables of the problem.

Linear Function: It is a mathematical expression that combines the variables to express goal. It may presents profits (maximize profit for minimize cost).

$$f(x_1, x_2) = c_1 x_1 + c_2 x_2 \tag{2}$$

Where c_1, c_2 are the coefficients of the objective function and x_1, x_2 are the variables of the problem.

Problem Constraint: Problem constraints are the mathematical expressions that combine the variables to express limits on the possible solutions.

$$a_{11}x_1 + a_{12}x_2 \leq b_1 \quad (3)$$

Where, a_{11}, a_{12} are the coefficients of the decision variables, x_1, x_2 are the variables of the problem and b_1 is the constraint value.

LP has generally two types of methods: graphical and simplex. Both methods can be applied on minimization and maximization problems. Graphical method is applicable only for solving an LP problem having two variables in its constraints. George Dantzig, a member of the U.S. Air Force, developed the Simplex method of optimization in 1947 in order to provide an efficient algorithm for solving programming problems that had linear structures. The simplex method is effective iterative method and is used in solving LP problem, containing several variables and constraints.

2.1. Simplex Algorithm

The Simplex algorithm is also often referred to as the Simplex method, is an iterative procedure for solving a class of problems. A desirable property of an algorithm is that it is finite, meaning that it is guaranteed to generate a solution to any problem instance in the specified class in a finite number of iterations. In each iteration of the Simplex method, the primary algebraic task is to transform, using Gaussian elimination, the constraint equations from a given configuration to a new configuration that corresponds to the next basic feasible solution. Such transformations are to be repeated many times in the course of the solution of a problem. It is therefore desirable to execute them in a manner that is as efficient as possible.

The steps of the Simplex algorithm are as follows:

1. Add slack variables to change the constraints into equations and write all variables to the left of the equal sign and constants to the right.
2. Write the objective function with all nonzero terms to the left of the equal sign and zero to the right. The variable to be maximized must be positive.
3. Setup the initial simplex tableau by creating an augmented matrix from the equations, placing the equation for the objective function last.
4. Determine a pivot element and use matrix row operations to convert the column containing the pivot element into a unit column.
5. If negative elements still exist in the bottom row, repeat Step 4. If all elements in the bottom row are positive, the process has been completed.
6. When the final matrix has been obtained, determine the final basic solution. This will give the maximum value for the objective function and the values of the variables where this maximum occurs.

2.2. BIG-M Method

BIG-M method is the variation of the simplex method designed for solving problems that involve "greater-than" constraints as well as "less-than" constraint. It refers to a large number associated with the artificial variables, represented by the letter "M". Firstly, BIG-M finds a basic feasible solution by adding "artificial" variables to the problem. The objective function of the original LP is modified to ensure that the artificial variables are all equal to 0 at the conclusion of the algorithm. The steps of the BIG-M method are:

1. Modify the constraints of LPP so that the RHS of each constraints is nonnegative and identify each constraints as a ≤ , ≥ , or = constraint.
2. Convert each inequality constraint to standard form by adding a slack variable for ≤ constraints, and a surplus variable for ≥ constraints.
3. Add an artificial variable to the constraints identified as ≥ or = constraints and add also sign restriction $a_i \geq 0$.
4. Let M denote a very large positive number. Add $-Ma_i$ to each artificial variable for max problem objective functions and Ma_i to each artificial variable for min problem objective functions.
5. Eliminate all artificial variables in objective function and solve the transformed problem by the simplex method.

In this paper, BIG-M method with Simplex algorithm is applied to find the optimal solution of advertising media problem.

3. Advertising Media Problem

Business organization wants to advertise in various advertisement media $x_1, \ldots, x_n$ and has B kyat in funds available for advertising budget. Each advertising media defined maximum advertisements D to advertise if the advertisers want to advertise in their programs. Advertisement media can define their popularity ratio accordingly to the number of their audience in the media market.

The Business organization wants to know how to advertise in various media within their advertising budget in order to maximize the ratio of the audience that knows their product. The standard form of the advertising media problem for maximizing target audiences is:

Objective Function

$$MAX\ Z = a_1x_1 + a_2x_2 + \cdots + a_nx_n$$

Budget Limitation Constraint

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n \le B$$

Program Constraint

$$x_1 \le D_1$$
$$\vdots$$
$$x_n \le D_n$$

The standard form of the advertising media problem for minimizing advertising budget is:

Objective Function

$$MIN\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Audience Constraint

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n \ge A$$

Program Constraint

$$x_1 \le D_1$$
$$\vdots$$
$$x_n \le D_n$$

Where, a is the number of audiences ratio of each advertisement, x_1 ,…, x_n are number of advertisement in media (journals, magazine, newspapers) respectively, c is charges for advertisement, B is budget limit, A is audience constraint that want to reach by the advertiser, and D is maximum advertisements that can be advertise by the advertiser.

4. System Overview

This system is developed as web-based system for generic businesses whose are finding appropriate advertising media for improving their market. It allowed two types of privileges Administrator and Users.

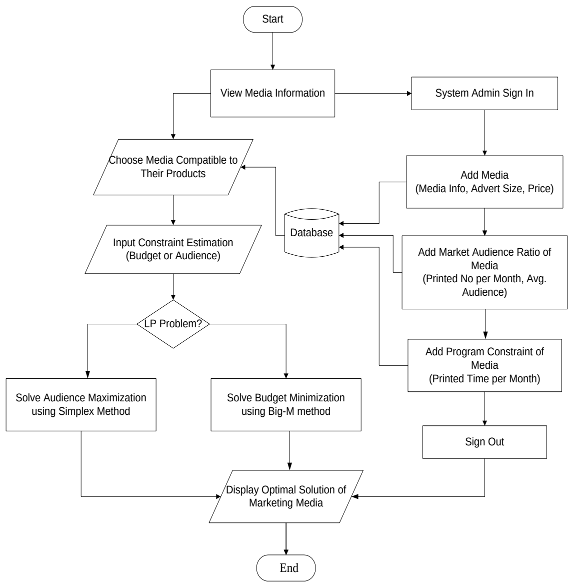

Figure 1. System Design

Administrator of this system set up media types and their costs, audiences for each media type and their advertising behaviors. According to the administrator setup, the advertising cost and audience ratios are shown as follow:

Table 1. Advertising Media Data

Media Name	Advertising Price	Avg. Audience	Printed Times/Month
Digital Life Journal	50000.00 Ks	90000	4
Living Color Magazine	30000.00 Ks	60000	1
The Time Newspapers	50000.00 Ks	60000	30

To choose efficient advertising media, users can choose media types by setting budget for advertising in order to reach their goals. Then, the system automatically formulated the standard form of user's advertising media problem. The system calculated the maximization problem of standard form using the Simplex method. The standard form of maximization is as showed in below.

User Input: B = 300000

Objective Function

$$\text{Maximize(audience)Z} = 90000x_1 + 60000x_2 + 60000x_3$$

Budget Limitation Constraint

$$50000x_1 + 30000x_2 + 50000x_3 \le 300000$$

Table 2. Initial Simplex Table of Maximization

C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
		Cj	**3**	**2**	**2**	**0**	**0**	**0**	**0**	
CB	**B**	**(=xB)**	**x1**	**x2**	**x3**	**s1**	**s2**	**s3**	**s4**	**Min Ratio**
0	s1	30	5	3	5	1	0	0	0	6
0	s2	4	1	0	0	0	1	0	0	4
0	s3	1	0	1	0	0	0	1	0	-
0	s4	30	0	0	1	0	0	0	1	-
z=0		zj	0	0	0	0	0	0	0	
		cj-zj	3	2	2	0	0	0	0	

Program Constraint

$$x_1 \leq 4$$
$$x_2 \leq 1$$
$$x_3 \leq 30$$

In the formulated standard form, x_1 is Digital Life Journal, x_2 is Living Color Magazine, and x_3 is The Time Newspapers. Then, the formulated standard form is simplified and transformed into Simplex tableau by creating augmented matrix from the standard form of advertising media problem. The initial Simplex table is shown in table 2.

Simplification of Advertising Media Problem (Maximization):

$$Z = 3x_1 + 2x_2 + 2x_3$$
$$5x_1 + 3x_2 + 5x_3 \leq 30$$
$$x_1 \leq 4$$
$$x_2 \leq 1$$
$$x_3 \leq 30$$

Standard Form of Audience Maximization:

$$3x_1 + 2x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = Z$$
$$5x_1 + 3x_2 + 5x_3 + s_1 = 30$$
$$x_1 + s_2 = 4$$
$$x_2 + s_3 = 1$$
$$x_3 + s_4 = 30$$

Then, the system determined the pivot element and used matrix row operations to convert the column containing the pivot element into the unit column. The row operation is repeated until all elements in the bottom row are positive and then final optimal solution is obtained. The final optimal solution of advertising media problem for audience maximization can be shown in:

The number of times to advertise:

Digital Life Journal = 4 times

Living Color Magazine = 1 time

The Time Newspapers = 1.4 = 1 time

The maximum audience of the advertising budget:

Max Z (Audience) = 16.8 * 30000 = 504000

After that, the system minimized the advertising budget of advertising media problem for optimal solution of target audiences using BIG-M method. The standard form of minimization is as shown in below.

Optimal Solution of Audience Maximization:

A = 504000

Objective Function

$$\text{Minimize}(cost)\ Z = 50000x_1 + 30000x_2 + 50000x_3$$

Audience Constraint

$$90000x_1 + 60000x_2 + 60000x_3 \geq 504000$$

Program Constraint

$$x_1 \leq 4$$
$$x_2 \leq 1$$
$$x_3 \leq 30$$

In standard form, BIG-M method added surplus and artificial variables for greater than constraints. The arbitrarily large number M is multiplied the artificial variables in the objective function for greater than constraints. Then, the formulated standard form is simplified and transformed into Simplex tableau by creating augmented matrix from the standard form of advertising media problem. The initial Simplex table is showed in table 3.

Simplification of Advertising Media Problem (Minimization):

$$Z = 5x_1 + 3x_2 + 5x_3$$
$$15x_1 + 10x_2 + 10x_3 \geq 84$$
$$x_1 \leq 4$$
$$x_2 \leq 1$$
$$x_3 \leq 30$$

Standard Form of Budget Minimization:

$$5x_1 + 3x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + MA_1 = Z$$
$$15x_1 + 10x_2 + 10x_3 - s_1 + A_1 = 84$$
$$x_1 + s_2 = 4$$
$$x_2 + s_3 = 1$$
$$x_3 + s_4 = 30$$

Table 3. Initial Simplex Table of Minimization

C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
		Cj	**5**	**3**	**5**	**0**	**0**	**0**	**0**	**M**	
CB	**B**	**(=xB)**	**x1**	**x2**	**x3**	**s1**	**s2**	**s3**	**s4**	**A1**	**Min Ratio**
M	A1	84	15	10	10	-1	0	0	0	1	5.6
0	s2	4	1	0	0	0	1	0	0	0	4
0	s3	1	0	1	0	0	0	1	0	0	-
0	s4	30	0	0	1	0	0	0	1	0	-
z=84M		Zj	15M	10M	10M	-M	0	0	0	M	
		cj-zj	5-15M	3-10M	5-10M	M	0	0	0	**0**	

Then the system calculated the simplex table using matrix row operation repeated until all elements in the bottom row are negative and then final optimal solution is obtained. The final optimal solution of advertising media problem for budget minimization is shown in below:

The number of times to advertise:
Digital Life Journal = 4 times
Living Color Magazine = 1 time
The Time Newspapers = 1.4 = 1 time
The minimize budget of the advertising media problem:
Min Z (Budget) = 30 * 30000 = 300000

5. Conclusion

Linear Programming is suitable technique to apply to the media selection problem. In applying Linear Programming to media selection problem, it has been observed that the constraint structure must be imposed which is essentially judgmental. The constraints are fundamental judgments about the nonlinearity of response. This system can help to find out the best or optimal advertising media within constraint set. BIG-M method with Simplex algorithm is used to solve the linear programming in optimal way. This system will support in finding optimal way for advertising to marketer.

References

[1] E. F. Robertson J. J. O'Connor. George Dantzig. http://www.history.mcs.standrews.ac.uk/Biographies/ Dantzig George.html, University of St. Andrews,2003.

[2] G. B. Dantzig, Maximization of a linear function subject to linear inequalities, in T. C. Koopmans (ed.), Activity Analysis of Production and Allocation, John Wiley & Sons, New York, 1951, pp. 339–347.

[3] J K SHARMA "Operation Research Theory and Applications" , CS 401.

[4] R. G. Bland New finite pivoting rules for the simplex method, Mathematics of Operations Research 2 (1977) 103–107.

[5] Williams, H. P.: Model Building in Mathematical Programming, 3d ed., Wiley, New York, 1990.

[6] Wayne L. Winston. Operations Research: Applications and Algorithms. Duxbury Press, fourth edition, 2003.